Adaptive nichtlineare Regelung mittels RLS und MPC (C/GMRES)

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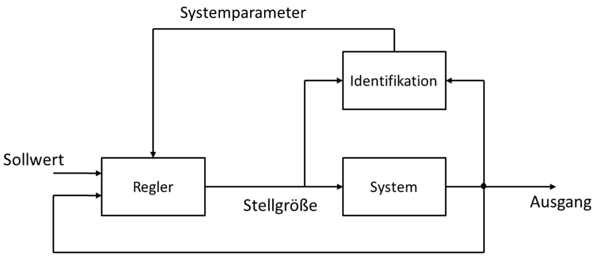
Einführung

- Motivation
  - Optimale Regelung nichtlinearer Systeme
  - Systemverhalten ändert sich über der Zeit
- Adaptiver Regelkreis
  - Identifikation: Recursive Least Squares (RLS)
  - Regler: Nonlinear Model Predictive Control (NMPC) nach C/GMRES

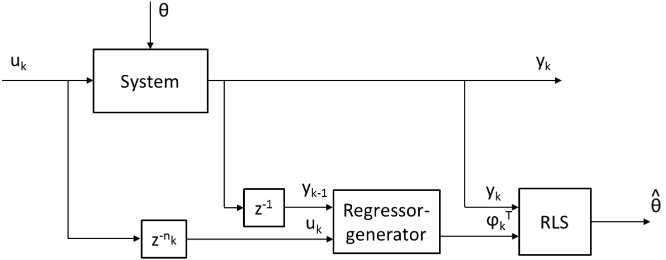


Recursive Least Squares

- Modellstruktur
  - Polynomiale NARX Modelle

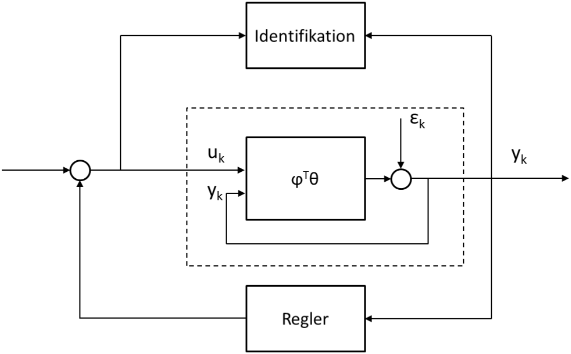
$$
\begin{aligned}
y_k = {} & \theta_0 + y_{k-1}\theta_1 + y_{k-2}\theta_2 + u_{k-1}\theta_3 + y_{k-1}y_{k-2}\theta_4 + \\
& y_{k-1}u_{k-1}\theta_5 + y_{k-2}u_{k-1}\theta_6 + y_{k-1}^2\theta_7 + y_{k-2}^2\theta_8 + u_{k-1}^2\theta_9 + \epsilon_k \\
= & \varphi_k^T \theta + \epsilon_k
\end{aligned}
$$

  - Regressorselektion zur Vermeidung von Overfitting
- Exponentielles Vergessen
- Implementierung



Closed-Loop-Identifikation

- Probleme
  - Unzureichende Anregung
  - Korrelation der Störung ϵ_k mit den Regressoren
- Abhilfe
  - Dithersignal (Zusätzliches Anregungssignal)



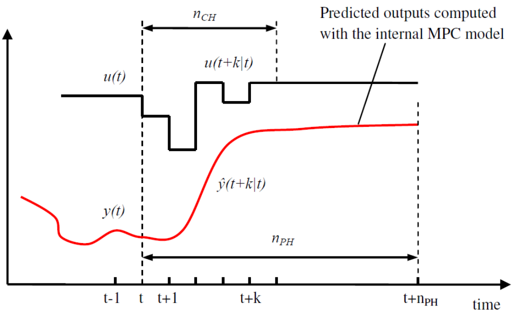
Nonlinear Model Predictive Control

- Modell

$$x_{k+1} = f(x_k, u_k, p_k)$$

- Minimieren der Kostenfunktion

$$J = \varphi(x^u(k+K, x_k), p_{k+K}) + \sum_{i=k}^{k+K-1} L(x_i^u, u_i, p_i)$$



C/GMRES-MPC

- Prinzip der C/GMRES-Methode
  - Euler Lagrange Formulierung

$$H = \lambda^T f + L$$

$$\frac{\partial H}{\partial u} = 0, -\lambda^T = \frac{\partial H}{\partial x}, \lambda_K^T = \frac{\partial \varphi}{\partial x}$$

$$F(U) = \begin{bmatrix} \frac{\partial H}{\partial u_1} \\ \vdots \\ \frac{\partial H}{\partial u_K} \end{bmatrix} = 0, U = [u_1, \ldots, u_K]$$

  - Continuation method

$$\dot{F}(U) = \frac{\partial F}{\partial U}\dot{U} + \frac{\partial F}{\partial x}\dot{x} + \frac{\partial F}{\partial t}$$

$$\dot{F} = A_S F$$

$$\dot{U} = \frac{\partial F^{-1}}{\partial U}(A_S F - \frac{\partial F}{\partial x}\dot{x} - \frac{\partial F}{\partial t})$$

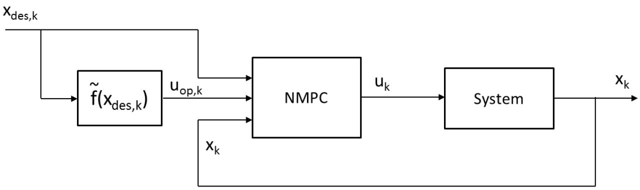
Diskreter Input/Output-C/GMRES-MPC

- Modell

$$x_{k+1} = f(x_k, \ldots, x_{k-n_x}, u_{k-n_k}, \ldots, u_{k-n_k-n_u}, p_{k-n_{pk}}, \ldots, p_{k+1-n_{pk}-n_p})$$

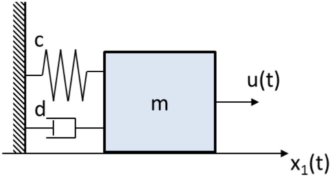
- Implementierung
  - Stationäre Referenz $x_{des,k}$
  - Stationärer Eingangsoffset $u_{op,k}$

$$x_{des,k} = f(x_{des,k}, u_{op,k}, p_k) \rightarrow u_{op,k} = \tilde{f}(x_{des,k}, p_k)$$



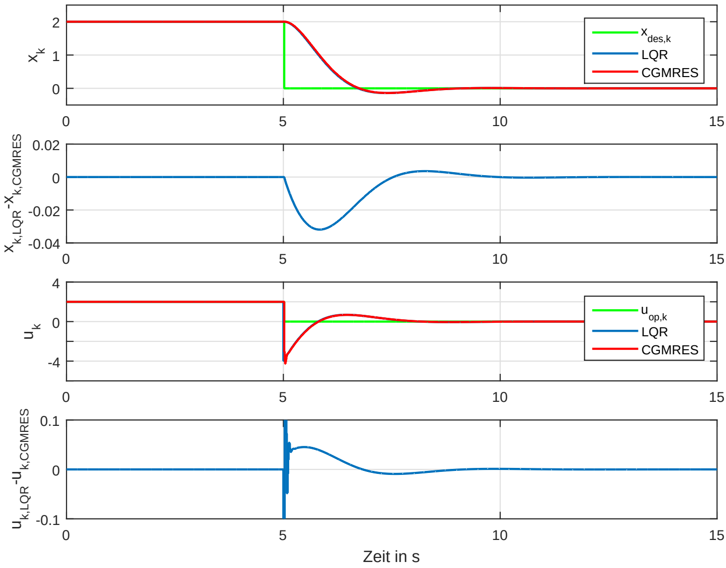
Analyse der C/GMRES-Lösung

- Regelung eines linearen Federmassenschwingers mit $m = 1\,\text{kg}$, $d = 1\,\text{Ns/m}$ und $c = 1\,\text{N/m}$



$$x_{k+1} = x_k(2 - T_s) - x_{k-1}(1 - T_s + T_s^2) + T_s^2 u_{k-1};\ T_s = 0.01\,\text{s}$$

- C/GMRES-MPC
  - $L = \tilde{Q}(x_k - x_{des,k})^2 + \tilde{R}(u_k - u_{op,k})^2$ mit $\tilde{Q} = 8T_s$, $\tilde{R} = T_s$
  - Prediction horizon $T_f = 10s$
- LQR
  - $Q = \text{diag}(8T_s, 10^{-6})$ und $R = T_s$



x_k
2
1
0
$x_{des,k}$
LQR
CGMRES
0
5
10
15
$x_{k,LQR}-x_{k,CGMRES}$
0.02
0
-0.02
-0.04
u_k
4
0
-4
$u_{op,k}$
$u_{k,LQR}-u_{k,CGMRES}$
0.1
-0.1
Zeit in s

Kombination von RLS und MPC

- Regelung eines nichtlinearen Federmassenschwingers mit unbekannten $m(t)$, $d(t)$ und $c(t)$

$$x_{k+1} = 2x_k - x_{k-1} - \frac{T_s^2 c_k}{m_k} x_{k-1}^3 - \frac{T_s d_k}{m_k}(x_k - x_{k-1}) + \frac{T_s^2}{m_k} u_{k-1} + \epsilon_k$$

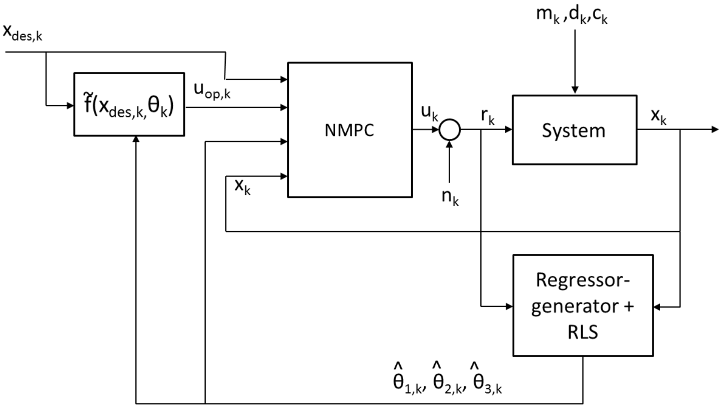
- Regressortransformation

$$x_{k+1} - 2x_k + x_{k-1} =$$

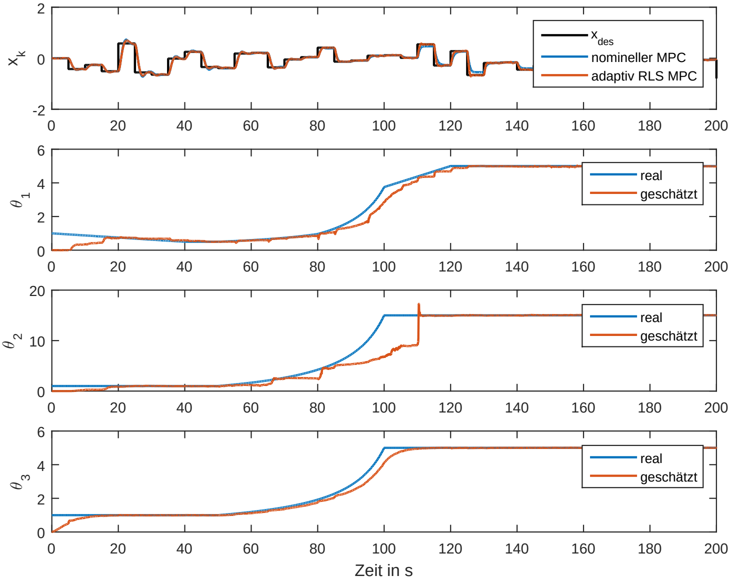
$$\begin{bmatrix} -T_s(x_k - x_{k-1}) & -T_s^2 x_{k-1}^3 & T_s^2 u_{k-1} \end{bmatrix} \cdot \begin{bmatrix} \theta_{1,k} \\ \theta_{2,k} \\ \theta_{3,k} \end{bmatrix} + \epsilon_k$$

- Simulationsparameter
  - $T_s = 0.01\,\mathrm{s}$
  - $\tilde{Q} = 8T_s$, $\tilde{R} = T_s$
  - Prediction horizon $T_f = 1\,\mathrm{s}$
  - Vergessensfaktor $\lambda = 0.997$
  - Dithersignal $n_k \sim \mathcal{N}(0, 0.01)$

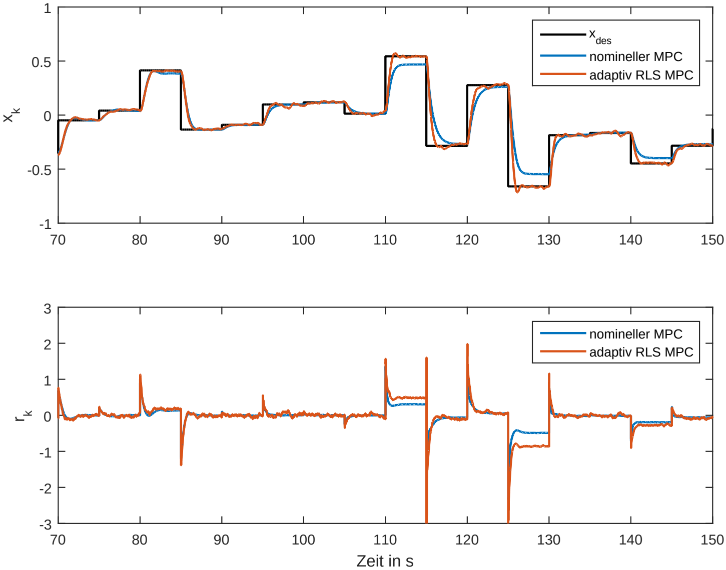
Kombination von RLS und MPC



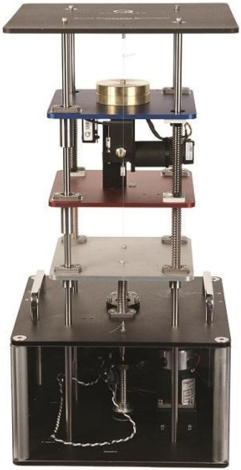
Kombination von RLS und MPC

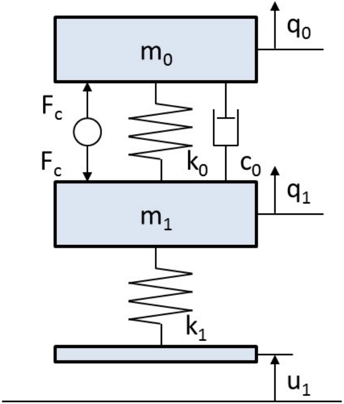


Kombination von RLS und MPC



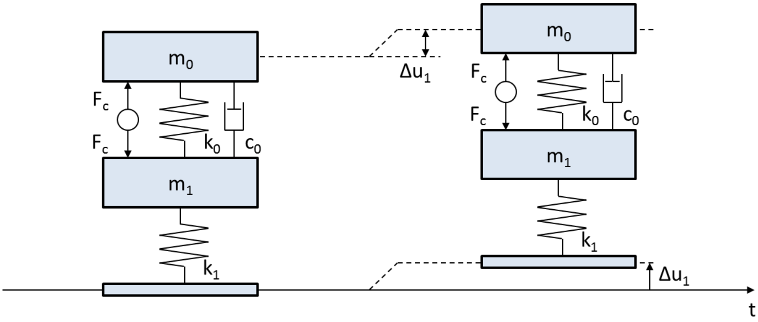
Reales System Viertelfahrzeug





Reales System Viertelfahrzeug

- Ziel: Regulierung der Geschwindigkeit $v_0(t)$ zu 0
- Umsetzung: Folgeregelung für die Vertikalposition $q_0(t)$ mit Referenz $u_1(t)$



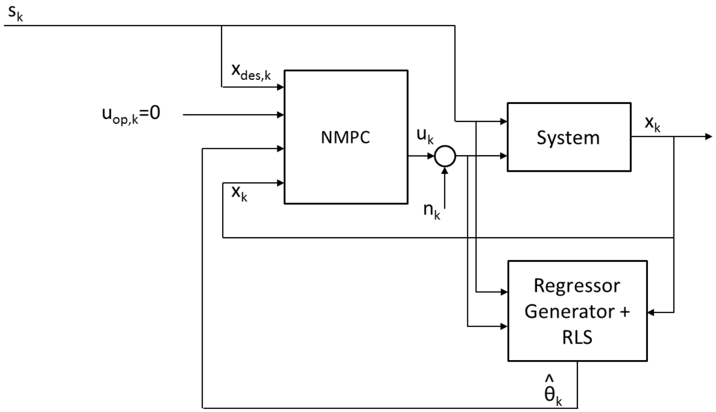
Reales System Viertelfahrzeug

- Modell

$$x_{k+1} = -\theta_1 x_k - \theta_2 x_{k-1} - \theta_3 x_{k-2} - \theta_4 x_{k-3} + \theta_5 s_k + \theta_6 s_{k-1} + \theta_7 s_{k-2} + \theta_8 s_{k-3} + \theta_9 u_k + \theta_{10} u_{k-1} + \theta_{11} u_{k-2} + \theta_{12} u_{k-3}$$

- Parameter
  - $T_s = 1\,\mathrm{ms}$
  - $\tilde{Q} = 10^8\, T_s,\ \tilde{R} = T_s$
  - Prediction horizon $T_f = 0.04\,\mathrm{s}$
  - Stellgrößenbeschränkung $\pm 15\,\mathrm{N}$
  - Vergessensfaktor $\lambda = 0.999$
  - Dithersignal $n_k = 0$

Reales System Viertelfahrzeug



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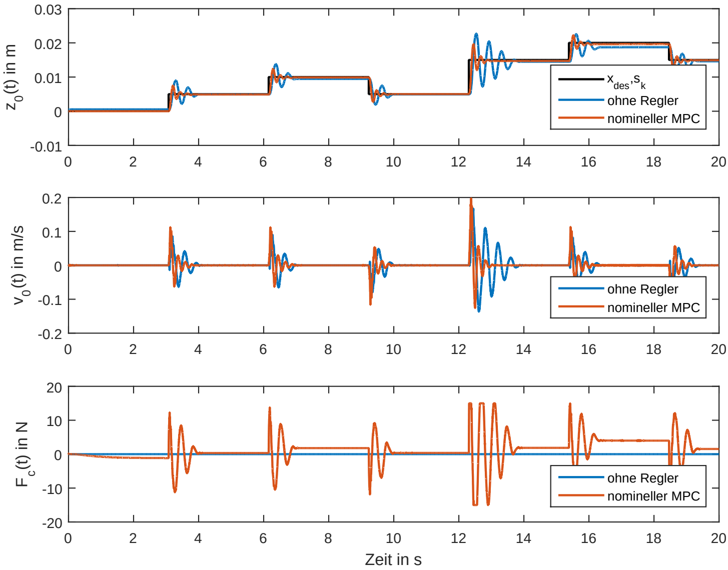

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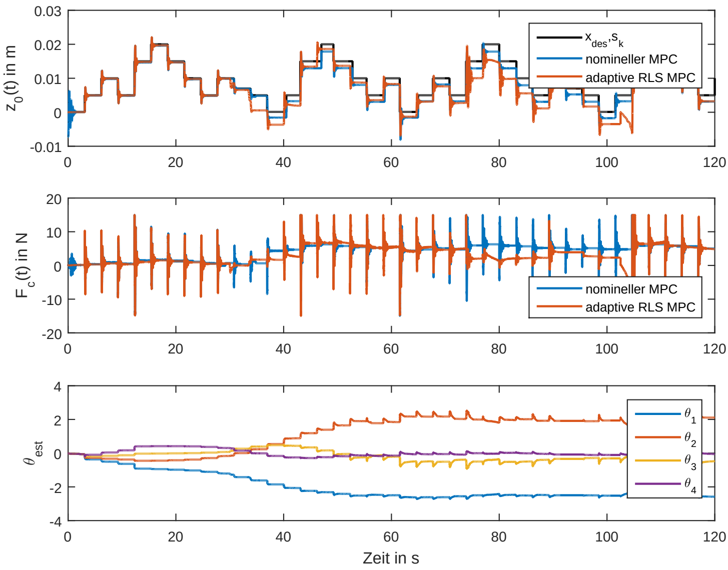
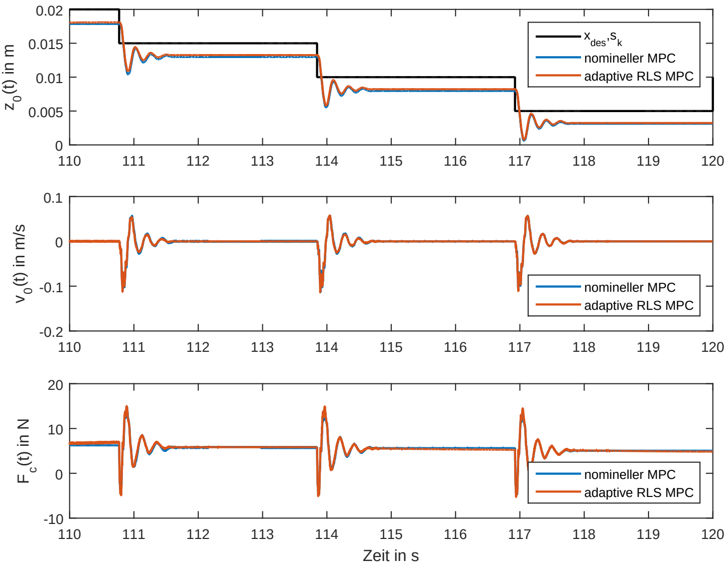


Reales System Viertelfahrzeug



Zusammenfassung

- Probleme der Identifikation
  - Unzureichende Anregung
  - Overfitting
- Kombination von RLS und C/GMRES-MPC
- Erhöhung der Performance