Institute for Design and Control of Mechatronical Systems

Diploma Thesis:

Tube Based Robust Model Predictive Control Polytopic and Ellipsoidal Approaches

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Abstract

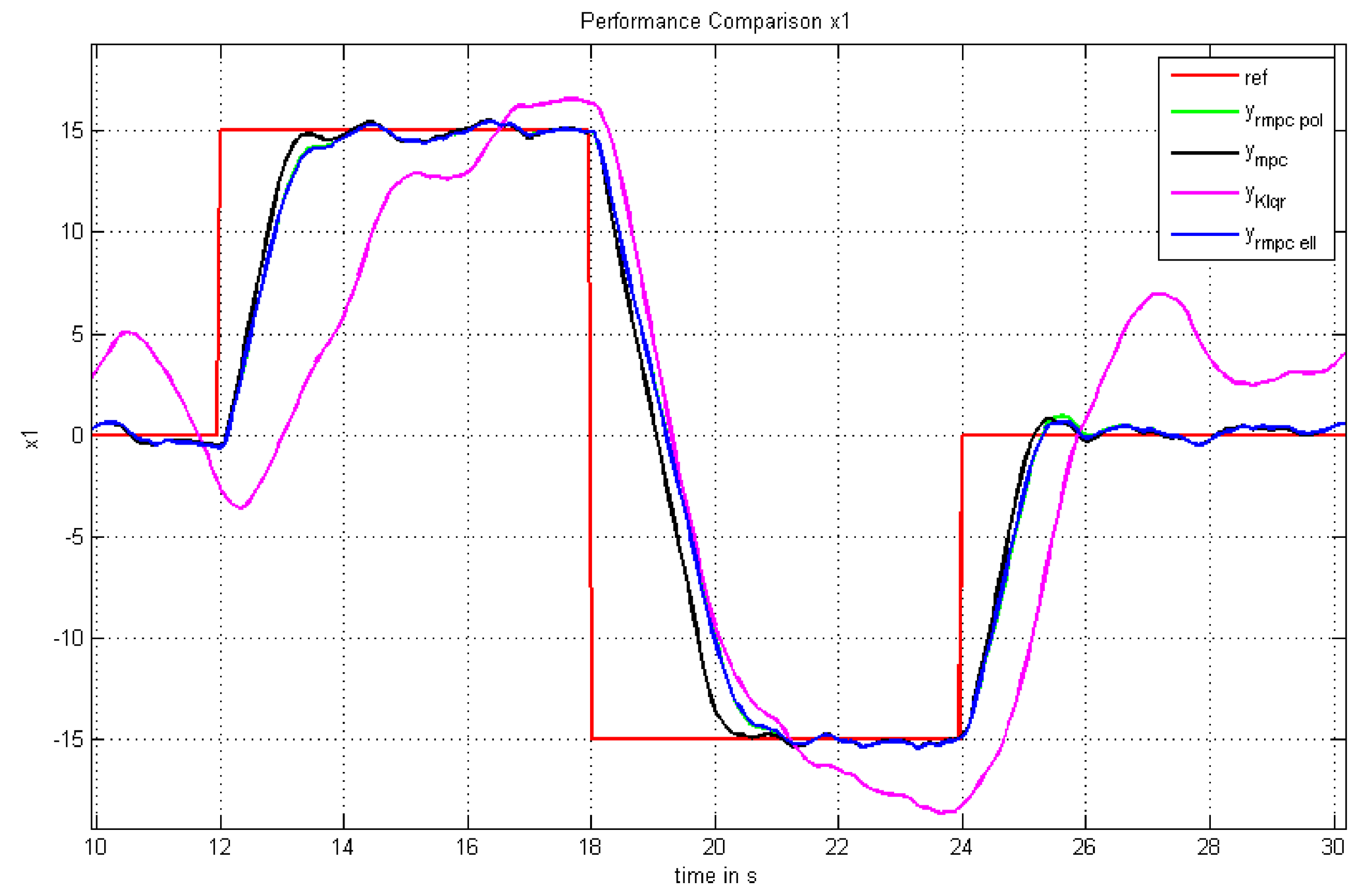
This thesis deals with the extension of the standard model predictive control (MPC) strategy, such that robust constraint satisfaction and stability in the presence of unknown but bounded external disturbances can be guaranteed. This leads to tube based robust model predictive control (TRMPC), in which the optimization problem is solved for the nominal system, i.e. the system without uncertainties. The uncertainties are taken into account by ensuring that the actual system state remains within a positive invariant vicinity of the nominal system state.

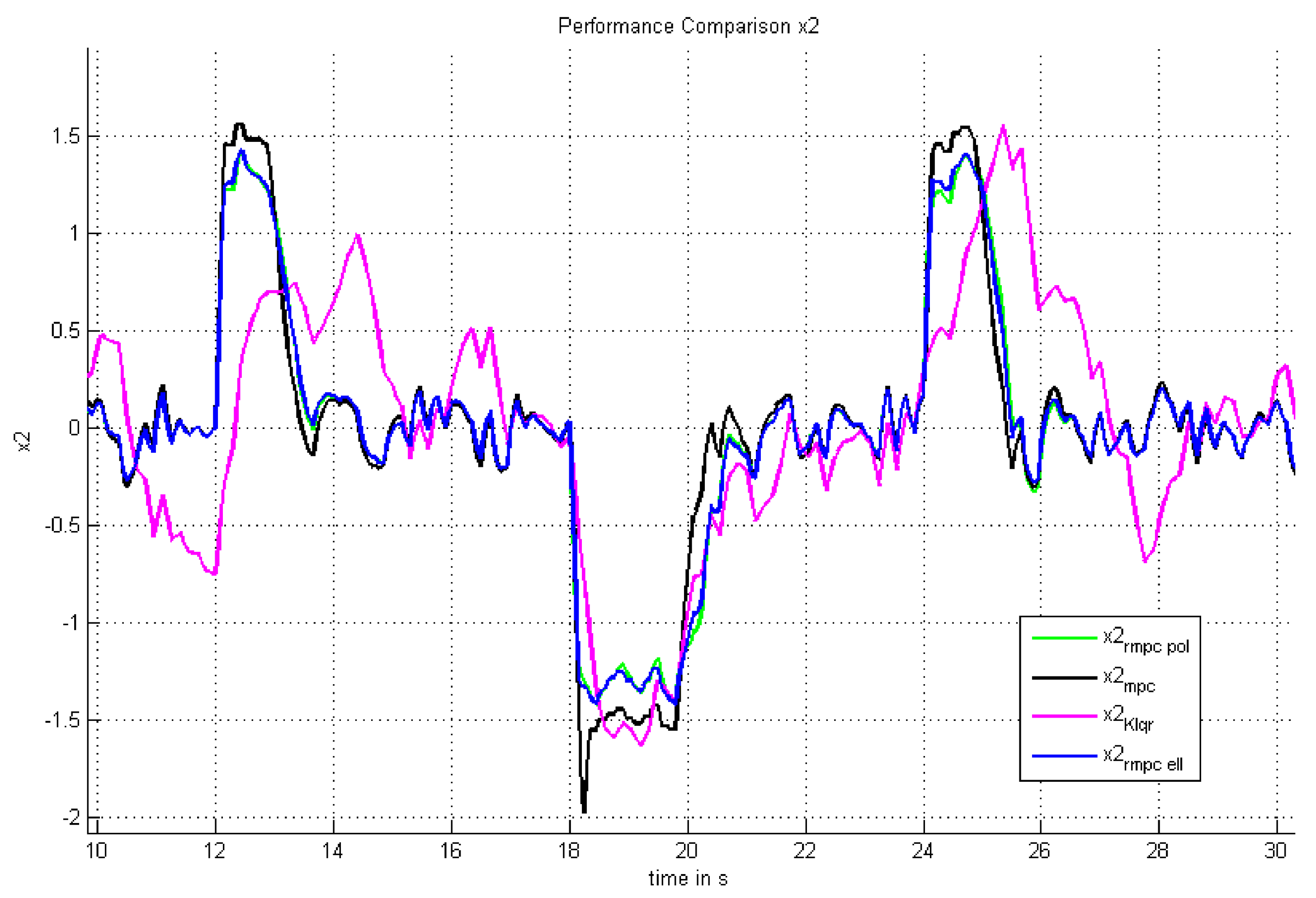
Introduction

Computational power is getting cheaper and cheaper and simultaneously more powerful and therefore possible applications for MPCs grow steadily.

One of the biggest advantages of MPCs is the easy implementation of (hard) constraints, but in real world applications model plant mismatches and disturbances are inevitable, and thus violations of the given constraints might occur during the operation of the plant. In the example below x2 should fulfill $|x2| \leq 1.5$, but this is violated in the case of the standard MPC (and state controller). Fortunately, TRMPCs are capable of coping with these problems and on top of that, the online computational complexity of TRMPCs is, in general, rather low when compared to other robust control strategies like min-max robust MPCs.

TRMPCs are based on appropriately tightened state and input constraints and two positive invariant sets, one for the deviation of the nominal and the actual system state and one for the predicted terminal state. These sets are usually represented by polytopes or ellipsoids.

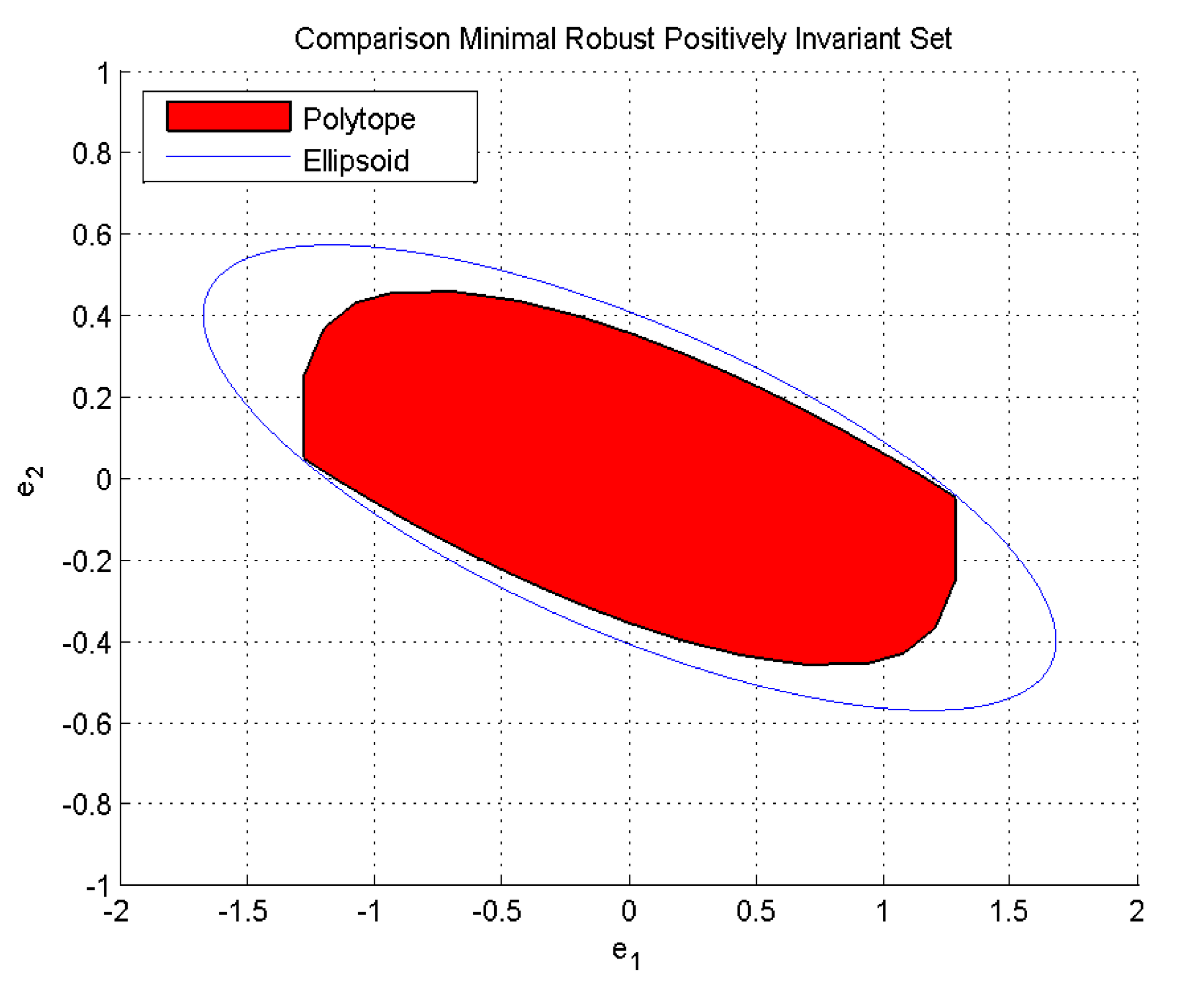




Comparison Polytopes & Ellipsoids

In terms of the optimization problem structure, the polytopic approach is favorable, because here it remains a simple QP (Quadratic Program), that is more tractable than a QCQP (Quadratically Constrained QP), which the ellipsoidal approach results in.

One aspect that clearly favors the ellipsoidal approach is the fact, that the number of additional (quadratic) constraints only depends on the system order and therefore is fixed, whereas the number of constraints for the polytopic approach might grow beyond tractability. By introducing reasonable simplifications, e.g. using (rough) approximations of the polytopic positive invariant sets and removing certain constraints entirely, it was found that the number of constraints for the polytopic approach, too, only depends on the system order. This combined with the simpler structure (QP) puts the polytopic approach ahead in terms of online computational times. On the other hand, the ellipsoidal approach is less expensive in terms of offline computations, because only linear matrix inequalities (LMI's) have to be solved for this approach, whereas the polytopic approach might require a high number of computationally expensive Minkowski-Sums .



Limitations

In practical applications the proper estimation of disturbances is probably the most critical part in the design process of a TRMPC. If they are estimated too large, then the TRMPC might be too conservative to be of any use (e.g. unsatisfactory performance, or an unacceptably small set of reachable steady set points). On the other hand, if they are estimated too small, then constraint satisfaction and even robust stability might be violated. Actually, if the disturbances are rather unknown than uncertain, a standard MPC with soft constraints is a more reasonable choice.

On top of that, (linear) TRMPCs are not perfectly suited for non linear plants, as, obviously, the linearization of the plant is only limitedly valid and the positive invariant sets and the controller in general are based on the assumption of a linear model. Therefore the positive invariance of the sets is questionable and subsequently robust stability and robust constraint satisfaction can, strictly speaking, not be guaranteed.

Conclusions and Outlook

Assuming the plant and the disturbances are very well known, then a TRMPC can be tuned in a way that it provides high performance, without risking constraint violations, stability or infeasibility issues, but wrong assumptions may still lead to constraint violations or even instabilities.

Comparing the polytopic and the ellipsoidal approach, both have their respective benefits and consequently it depends on the application, which approach is more beneficial. Furthermore, it can be expected that their respective drawbacks, i.e. larger offline computational complexity (polytopes) and larger online computational complexity (ellipsoids) will be reduced by future algorithms and solvers.