Institute for Design and Control
of Mechatronical Systems

Diploma Thesis:

Nonlinear Approximate Optimal Control Using C/GMRES

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Abstract

In the resent years it became popular to solve control problems by using optimization. The most widespread form of such control strategies is model predictive control or MPC. The input to the plant is calculated in a way, that a cost function is minimized over a certain horizon of time. In this thesis a known method for solving an optimization problem considering continuous-time state space models is transformed into a method to solve an optimization problem, bounded by discrete-time descriptions of the system, described by a state space model or an input-output model. The developed optimization routines are applied to a rather simple toy example to demonstrate their functionality and to the airpath of a passenger car's diesel engine on a test bench.

Introduction

Nonlinear continuous-time models are basically given in the form

$$\dot{x}(t) = f(x(t), u(t), p(t)).$$

bounded by equality constraints

$$C(x(t), u(t), p(t)) = 0.$$

This system should be controlled by the control inputs $u(t)$, minimizing a performance index over a receding horizon

$$J = \varphi(x^u(t+T, x(t)), p(t+T)) + \int_t^{t+T} L(x^u(\tau, t, x(t)), u(\tau), p(\tau))\,d\tau.$$

This optimization problem is solved numerically by using both, the continuation method and the GMRES algorithm as shown by Prof. Ohtsuka.
In several applications a physical modeling approach is quite complex, so data-based modeling approaches are used. These models are in most cases discrete-time I/O models. In this thesis the airpath of a turbocharged diesel engine is considered as an example for such a complex system. The physical modeling of the airpath is usually done via mean-value-models neglecting the wave propagation in the airpath, because this would result in a too complex model for control tasks. Additionally, for a diesel engine it is hard to bring up a model of the combustion process itself, because there is no certain time instant for the start of the ignition process.In literature only a hand full of solvers for real-time nonlinear optimal control and even less which can directly deal with discrete-time systems are described. To be able to deal with such discrete-time problems, the C/GMRES method is modified to deal with discrete-time domain state space and I/O models.

Derivation of the Necessary Equations

It is necessary to bring up equations for both, discrete-time state space and I/O model.
The main idea is that these equations can be derived by the principles of static optimization using the Lagrange-formalism.
This results in

$$H = \lambda_k^T f(x_k, u_k, p_k) + L(x_k, u_k, p_k) + \mu^T C(x_k, u_k, p_k)$$

$$\partial H / \partial u_k = 0, \quad \partial H / \partial x_k = 0$$

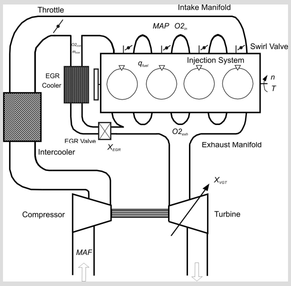
for the state space approach and in

$$0 = \partial L / \partial u_k + \mu_k^T \partial C / u_k + \sum_{i=k+n_k}^{k+n_k+n_b} \lambda_{i+1}^T \, \partial P(u_{i-nk}, \ldots) / u_k, \quad 0 = \partial L / \partial x_k + \mu_k^T \partial C / x_k + \sum_{i=k-1}^{k+n_a} \lambda_{i+1}^T \, \partial (P(x_{i-1}, \ldots) - x_{i+1}) / u_k$$

for the I/O model, using $x_k = P(x_{k-1}, \ldots, u_{k-nk}, \ldots, p_{k-np}, \ldots)$.

Airpath Control

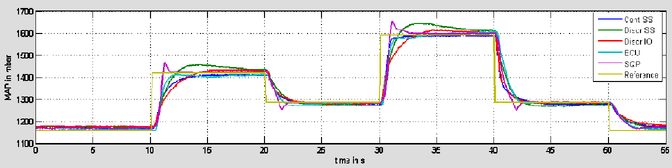
To show the functionality of the algorithms in practice, they were applied to the already existing model of an airpath of a diesel engine. The controllable inputs to the model are the position of the EGR and the VGT, the outputs the MAP and the MAF.
Additionally, the measured disturbances are the injected fuel amount and the actual speed of the engine.
With this model both, simulation and practical test on the testbench are performed.

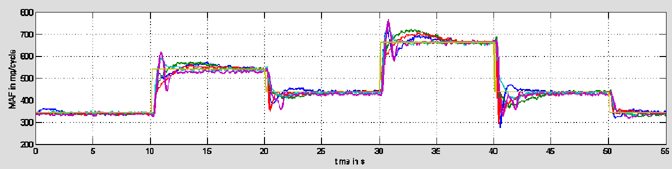


Measurement Results

To test the algorithms in practice, an artificial trajectory was tracked and the exhausts were measured. The tracking performance is Measured in the L2-norm.

Controller	e_{MAP}	e_{MAF}
ECU	55.75	29.20
continuous-time state space	49.54	31.00
discrete-time state space	57.34	32.73
discrete-time I/O model	53.88	31.43
SQP	52.00	35.81





Additionally, a part of the FTP-cycle was used as reference. The results, especially of the I/O model based NMPC seem to be promising for further work on reducing exhausts by improvement of the airpath controller performance.

Controller	e_{MAP}	e_{MAF}	CO	Opacitiy	NO	NOX	THC
ECU	72.08	96.14	1	1	1	1	1
continuous-time state space	117.39	67.24	2.086	1.405	0.982	0.833	1.236
discrete-time state space	80.03	54.45	1.288	1.385	0.735	0.739	1.154
discrete-time I/O model	69.42	55.72	1.060	1.373	0.749	0.811	0.955

Conclusions and Outlook

In conclusion of this thesis, the proposed controllers work well, both, in simulation and practice and the results are comparable. The controllers might work better, if they are tuned till the performance is maximal, but this was not the main point of this thesis. It is a great success to show, that even with higher sample time the controller have the sampe performance as the ECU so the assumption can be made, that with reduction of Ts the performance will increase. But on the other hand it is important to consider that a higher sample rate will increase the workload during the prediction, because the prediction horizon has to be long enough to include all important parts of the dynamics of the system. All in all a set of algorithms was developed, which allows the application of the C/GMRES method for nonlinear model predictive control to the models, resulting of the most common modeling strategies.