Institute for Design and Control of Mechatronical Systems

LCM — LINZ CENTER OF MECHATRONICS GMBH

Diploma Thesis:

Nonlinear Identification: Extended State-Affine Systems

Development of Regularization Methods and Investigation on different Examples

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Abstract

The aim of this thesis is to investigate the nonlinear identification using *Extended State Affine Systems* (ESA systems). In the first part of this thesis, the basics of the ESA systems are described. Furthermore, it is shown how data equations can be obtained for different approximation approaches. The author focuses the exact identification and its approximations. Also, the notation of the system class is described.
Another part is the development of regularization methods. To solve the identification problem it is necessary to solve an *inverse problem*. In the case of ESA, it the problem is usually *ill posed*. Using the different regularization methods the solution of the inverse problem can be found in a numerically more stable way. Three different groups of regularization methods are developed. First, the *Tikhonov regularization* methods; second, the *truncation regularization* methods and finally the *iterative methods*. For each group, different possibilities for *parameter choice rules* are described. At the end of this part, the presented methods are compared and evaluated. Hereby, the relevance of the *Kernel method* is investigated.
The last part of the thesis deals with the comparison of the ESA identification with other identification methods. Therefore, two other methods are used: the *ARMAX* identification and the *Neural Networks*. Under these conditions, different examples will be investigated. Several validation values are introduced and defined to compare the various methods on the examples. The extrapolation behaviour of the different methods is important. Furthermore, the examples are chosen in a way using a linear, a weak nonlinear and a nonlinear example. Also, the nonlinear example is compared to results in the literature.
Finally, advantages and disadvantages of the ESA identification are stated.

Introduction: Nonlinear ESA Identification

System description:

$$x(j+1)=Ax(j)+Nf\left(u(j),Cx(j)\right)\otimes x(j)+Bu(j)+Mg\left(u(j)\right)$$
$$y(j)=Cx(j)$$

Advantages:

- good approximation capabilities (based on fading memory assumption)
- state-space model
- includes several other model classes (bilinear systems, Hammerstein systems, ...)

Estimating Θ_X:

Starting from the Data Equation:

$$Y=\left[\Theta_X,\Theta_{F\odot X},\Theta_U\right]\begin{bmatrix}\Phi_X\\ \Phi_{F\odot X}\\ \Phi_U\end{bmatrix}+\Xi$$

Solving the following Least Squares Problem:

$$\left[\hat\Theta_X,\hat\Theta_Z\right]=\arg\min_{\Theta}\left\|Y-\left[\Theta_X,\Theta_Z\right]\begin{bmatrix}\Phi_X\\ \Phi_Z\end{bmatrix}\right\|_2^2$$

Estimating X:

From the measurement data:

$$\Gamma X\approx\Theta_X\Phi_X$$

Using the SVD:

$$\Gamma X=\left[U_n,U_0\right]\begin{bmatrix}S_n&0\\0&S_0\end{bmatrix}\begin{bmatrix}V_n^T\\V_0^T\end{bmatrix}$$
$$\Gamma=U_nS_n^{1/2}T$$
$$X=T^{-1}S_n^{1/2}V_n^T$$

Estimating System Matrices:

Solving the following Least Squares Problems:

$$\hat C=\arg\min_{C}\left\|Y_j-C\hat X_j\right\|_2^2$$

$$\left(\hat A,\hat N,\hat B,\hat M\right)=\arg\min_{A,B,N,M}\left\|\hat X_{j+1}-[A,N,B,M]\begin{bmatrix}\hat X_j\\ F\left(\hat C\hat X_j,U_j\right)\odot\hat X_j\\ U_j\\ G(U_j)\end{bmatrix}\right\|_2^2$$

Regularization Methods and Kernel Method

Following equation has to be solved to obtain Θ

$$Y=\Theta\Phi+\Xi$$

Appearing Problems

- ill-conditioned problems:
  - exploding dimension with increasing block size
  - inadequate persistent excitation
- under-determined problems:
  - not enough measurement data for larger block sizes

Solution Approaches

- Regularization Methods
  - to find better solutions for ill-conditioned problems
- Kernel Methods
  - to enable dealing with under-determined systems

Regularization Methods

- Tikhonov Regularization
- Tikhonov Regularization with Total Least Squares
- Truncated SVD
- Truncated TLS
- Iterative Regularization

Tikhonov-Regularization

finding a better conditioned solution near the ill-conditioned solution

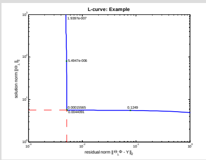
$$\Theta_\lambda=\arg\min_{\Theta}\left\{\left\|\Theta\Phi-Y\right\|_2^2+\lambda^2\left\|\Theta L^T\right\|_2^2\right\}$$

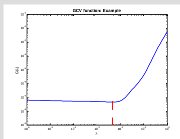
from this follows

$$\Theta_\lambda=Y\Phi^T\left(\Phi\Phi^T+\lambda^2L^TL\right)^{-1}$$

Tikhonov Regularization: Parameter Choice Rules

- Discrepancy Principal
- L-Curve Criterion
- Generalized Cross Validation (GCV)
- Quasi-Optimal Criteria
- Zero-Crossing Method





Kernel-Method

to enable dealing with under-determined systems

$$\Theta=\Psi\Phi^T$$

from this follows

$$\hat\Psi=Y\Phi^T\Phi\left(\Phi^T\Phi\Phi^T\Phi\right)^{-1}$$

Comparison of Regularization Methods

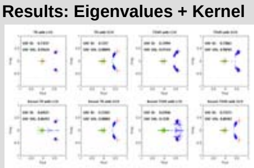
Parameters for Comparison

- VAF-value of the output

$$VAF=\max\left\{1-\frac{\operatorname{var}\left(y(j)-\hat y(j)\right)}{\operatorname{var}\left(y(j)\right)},0\right\}$$

- distance between the real and the determined eigenvalues

Results: Eigenvalues + Kernel

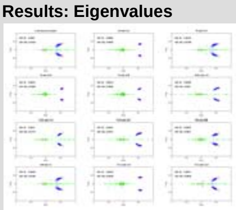


Regularization + Kernel: State Affine Example with SNR = 5dB, k = 4

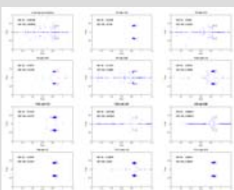
Conclusions

- use Kernel only in under-determined case
- regularization improves results
- best results with:
  - Tikhonov Regularization + L-Curve
  - Truncated Total Least Square + L-Curve

Results: Eigenvalues



Triangular ESA Example: SNR = 10dB

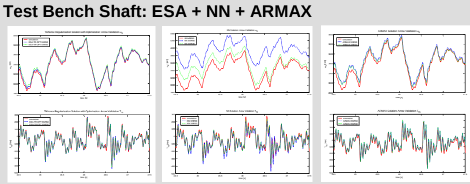


Continuous Extended State Affine System: SNR 15dB

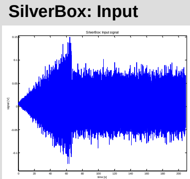
Comparison with other Methods

- Other Methods:
  - ARMAX identification
  - Neural Networks
- Used Examples
  - Linear: RotoFlex
  - Weak Nonlinear: Test Bench Shaft
  - Nonlinear: SilverBox

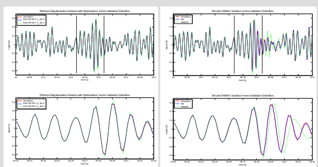
Test Bench Shaft: ESA + NN + ARMAX



SilverBox: Input



SilverBox: ESA + NN + ARMAX



Conclusions and Outlook

Conclusions

- **regularization improves the result compared to the least squares solution**
- **ESA-identification shows good extrapolation behaviors**
- **usage of constant term *g(u)* = 1 is useful**
- **lower computation effort for ESA than NN**
- **advantage of ESA for MIMO systems**
- **ESA uses no error model ➔ problems with high noise levels**
- **important that *f(u,y)* is chosen practical**
- **limitation of the matrix size (particular SVD)**
  - **limitation of system order**
  - **limitation of useable data points**
- **TTLS is not so sensible then TR**

Outlook

- **other mathematical solution than SVD**
- **error model**
- **maybe regularization ins general form**
- **further investigation on physical systems**
- **controller design for ESA systems**